Aspects of the Monster Tower Construction: Geometric, Combinatorial, Mechanical, Enumerative

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We survey the construction and use of the monster tower (also known as the Semple tower) in three distinct areas of mathematics.

Lecture 1: The Monster Tower (Kennedy) — This lecture will explain how three seemingly different situations lead to the same construction:

1. Compactifying curvilinear data (algebraic geometry)

2. Studying Goursat distributions (differential geometry)

3. Analyzing a truck with trailers (dynamics and control theory)

Lecture 2: Combinatorial Aspects (Colley) — We explain a natural system of coordinate charts on the monster space. We show how to lift (prolong) a curve in the base into the tower. We explain a natural coarse stratification of the monster, catalogued by a simple system of code words.

Lecture 3: Mechanical Aspects (Kennedy) — A version of the monster tower construction creates the natural configuration space for a truck with trailers. We explain the model and survey some important features, including Lie brackets of its basic vector fields, its singular configurations, and its dynamics.

Lecture 4: Enumerative Aspects (Colley) — We begin with an introduction (via examples) to the subjects of enumerative geometry and intersection theory. Specializing to the enumeration of contacts between plane curves, we present a strategy for counting such contacts and illustrate it with a quadruple contact formula we once proved. The ideas behind this formula lead to a discussion of the orbits of the monster space, and to the idea of appropriately lifting a family of curves.