Workshop on Nash Blow-up and Semple Tower, II

3–7 June 2019 Leuven (Belgium)

—Short courses—

Susan Colley & Gary Kennedy :
"Aspects of the Semple/Monster Tower:
Geometric, Combinatorial, Mechanical, and Enumerative"

Roi Docampo :
"Jet schemes and Nash–Semple modifications"

Erwan Rousseau :
"Generalized Demailly–Semple jet bundles"

—Organizers—

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—Sponsors—

KU Leuven · Institut Universitaire de France · Institut de Mathématiques de Marseille
GDR 'Singularités et applications' · GDR 'Géométrie Algébrique - Géométrie Complexe'

• Book of Abstracts •

Aspects of the Monster Tower Construction: Geometric, Combinatorial, Mechanical, Enumerative

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We survey the construction and use of the monster tower (also known as the Semple tower) in three distinct areas of mathematics.

Lecture 1: The Monster Tower (Kennedy): This lecture will explain how three seemingly different situations lead to the same construction:
(1) Compactifying curvilinear data (algebraic geometry)
(2) Studying Goursat distributions (differential geometry)
(3) Analyzing a truck with trailers (dynamics and control theory)

Lecture 2: Combinatorial Aspects (Colley): We explain a natural system of coordinate charts on the monster space. We show how to lift (prolong) a curve in the base into the tower. We explain a natural coarse stratification of the monster, catalogued by a simple system of code words.

Lecture 3: Mechanical Aspects (Kennedy): A version of the monster tower construction creates the natural configuration space for a truck with trailers. We explain the model and survey some important features, including Lie brackets of its basic vector fields, its singular configurations, and its dynamics.

Lecture 4: Enumerative Aspects (Colley): We begin with an introduction (via examples) to the subjects of enumerative geometry and intersection theory. Specializing to the enumeration of contacts between plane curves, we present a strategy for counting such contacts and illustrate it with a quadruple contact formula we once proved. The ideas behind this formula lead to a discussion of the orbits of the monster space, and to the idea of appropriately lifting a family of curves.

References.

- Montgomery, R. and Zhitomirskii, M., 2001, July. Geometric approach to Goursat flags. In Annales de l'Institut Henri Poincare (C) Non Linear Analysis (Vol. 18, No. 4, pp. 459-493). Elsevier Masson.
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- Bravo-Doddoli, A. and García-Naranjo, L.C., 2015. The dynamics of an articulated n-trailer vehicle. Regular and Chaotic Dynamics, 20(5), pp.497-517.
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Jet schemes and Nash–Semple modifications

Roi Docampo

University of Oklahoma – United States

The arc space of an algebraic variety is a geometric object parametrizing infinite-order jets of curves mapping to the variety. Similarly, the n-th jet scheme of the variety parametrizes n-th order jets of curves. Jet schemes and arc spaces have been used frequently in recent years, mainly due to their prominent role in the theory of motivic integration, and to their usefulness for the control of invariants of singularities in the context of the minimal model program. Nash-Semple modifications appear naturally in the study of the geometry of jet schemes and arc spaces. In these lectures I will give an introduction to the basic theory of arc spaces with emphasis in exhibiting the connections with Nash-Semple modifications and related constructions. In particular, I will discuss recent developments on the differential structure of arc spaces, explain how discrepancies control the geometry of arc spaces (here is where Nash-Semple modifications first appear), and discuss Nash-Semple modifications of jet schemes.

References.

- de Fernex, T. and Docampo, R., 2017. Nash blow-ups of jet schemes. arXiv preprint arXiv:1712.00911.
- de Fernex, T. and Docampo, R., 2017. Differentials on the arc space. arXiv preprint arXiv:1703.07505.
- Oneto, A. and Zatini, E., 1991. Remarks on Nash Blowing-up. Rend. Sem. Mat. Univ. Pol. Torino Vol. 49, 1 (1991) ACGA - 1990.

Generalized Demailly–Semple jet bundles

Erwan Rousseau

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The goal of these lectures is to present the construction of the generalized Semple tower following Arrondo, Sols and Speiser, with applications to hyperbolicity and holomorphic foliations.

References.

- Arrondo, E., Sols, I. and Speiser, R., 1997. Global moduli for contacts. Arkiv för matematik, 35(1), pp.1-57.
- McQuillan, M., 1999. Holomorphic curves on hyperplane sections of 3-folds. Geometric Functional Analysis GAFA, 9(2), pp.370-392.
- Pacienza, G. and Rousseau, E., 2011. Generalized Demailly–Semple jet bundles and holomorphic mappings into complex manifolds. Journal de mathématiques pures et appliquées, 96(2), pp.109-134.

Preparatory talks

3264: Enumerative Algebraic Geometry of Conics

Ferran Dachs-Cadefau

Martin-Luther-Universität Halle-Wittenberg – Germany

This is a preparatory talk for Gary Kennedy and Susan Colley's course. The main goal is to introduce Enumerative Algebraic Geometry. In order to do so, we present the solution to the following problem posed by Jakob Steiner in 1848: "Given five conics in the plane, are there any conics that are tangent to all five? If so, how many are there?". In particular, to present the solution, 3264, we will need to introduce/recall some concepts as the moduli space, the blow-up or the Chow ring that will be useful in order to solve many other similar problems.

Holomorphic Morse inequalities to produce global sections.

Corinne Bedussa

KU Leuven – Belgium

A compact complex manifold is (Brody) hyperbolic if it does not contain non-constant entire curves. It is classically well known that hyperbolicity is related to the existence of symmetric differentials and their higher order generalization ("jet differentials"). In this tallk we will describe some classical and modern techniques to produce these differentials as global sections of vector bundles.

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p-measure hyperbolicity in complex geometry and analogues in algebraic geometry

Antoine Etesse

Université Aix-Marseille – France

The talk aims to present the notions of infinitesimal p-measure hyperbolicity and p-measure hyperbolicité (the integral version of the last) for a complex variety X (or an analytic space), which constitutes a natural extension of the Kobayashi hyperbolicity. We then expose the analogues of these notions in an algebraic setting (especially for the extremal case i.e. 1-hyperbolicity and n-hyperbolicity, where n=dim X), and we relate these two points of view. We finish with an algebraic condition (i.e. the ampleness of the pth exterior power of the cotangent bundle of a variety) wich ensures the p-hyperbolicity of a complex variety, and we will see examples (non explicit but generic) of projectives varieties that satisfy this condition.

Jet schemes and arc spaces

Lena Vos

KU Leuven – Belgium

As a preparation for Roi Docampo's course, this talk provides an introduction to the jet schemes and the space of arcs associated to a scheme X of finite type over a field. Topics that will be discussed are their defining equations, truncation maps and irreducible components. In particular, the main part, or main component if X is variety, of a jet scheme will be introduced. The theory will be illustrated with multiple examples.

Nash blow-ups of coherent sheaves

Tue Tran

KULeuven – Belgium

As a preliminary talk for the course of Roi Docampo, this talk introduces the notion of the Nash blow-up as blow-up of a sheaf of modules, which is builts on the theory of Grassmannians of sheaves. The properties of this blow-up will be introduced, including the fact that the Nash blow-up can relate to the blow-up with respect to a fractionary ideal.

Complementary talks

The dynamics of an articulated n-trailer vehicle.

Alejandro Bravo Doddoli

Universidad Nacional Autónoma de México – Mexico

We consider the dynamics of an articulated n-trailer vehicle that moves under its own inertia. Such a system consists of a leading car, or truck, that is pulling n trailers, like a luggage carrier in the airport. This system is a canonical example in nonholonomic motion. The nonholonomic constraints arise by assuming that each of the body in the convoy has a pair of wheels that prohibit motion in the direction perpendicular to them. The constraints define a Goursat distribution with the property that all the Goursat germs of corank $n+1$ are realized at its different points. I began this project with the hope to find consequences in the dynamics arising by the singularity of the distribution. I will explain some results.

The Green–Griffiths-Lang conjecture for hypersurfaces (via jet techniques) implies the Kobayashi conjecture (after Riedl-Yang)

Benoit Cadorel

Institut de Mathématiques de Toulouse – France

In a recent work, Riedl and Yang obtained a very short proof of the Kobayashi conjecture, which predicted the hyperbolicity of general projective hypersurfaces of large degree. Their idea is to use a clever trick to show that if the Green-Griffiths-Lang locus is proper for a general hypersurface of some dimension, then for some lower dimension, the very general hypersurfaces will be hyperbolic. The conjecture is then reduced to a theorem of Diverio-Merker-Rousseau ; more recent results of Demailly can be used to obtain a better bound on the degree. Also, their technique permits to show hyperbolicity of the complement of very general hypersurfaces of large degree, building up on the work of Darondeau.
